

**Charged Rotating Black Holes in Five Dimensional  $U(1)^3$  Gauged  $\mathcal{N} = 2$  Supergravity**M. Cvetič<sup>\*1</sup>, H. Lü<sup>†‡2</sup> and C.N. Pope<sup>†2</sup>*\*Department of Physics and Astronomy,  
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University of Science & Technology of China, Hefei, Anhui 230026, China***ABSTRACT**

We obtain the general solution for non-extremal 3-charge dilatonic rotating black holes in the  $U(1)^3$  gauged five-dimensional  $\mathcal{N} = 2$  supergravity coupled to two vector multiplets, in the case where the two rotation parameters are set equal. These solutions encompass all the previously-known extremal solutions, and, by setting the three charges equal, the recently-obtained non-extremal solutions of  $\mathcal{N} = 2$  gauged five-dimensional pure supergravity.

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Valuable tests of the AdS/CFT correspondence can be performed by taking charged black holes with non-zero cosmological constant as gravitational backgrounds [1, 2]. The black-hole charges play the rôle of R-charges in the dual field theory [3]. Furthermore, one can gain insight into the phase structure of the strongly-coupled dual field theory by studying the thermodynamic stability of the black-hole solutions, and the analogue of the Hawking-Page transition [3, 4, 5]. The first examples of non-extremal charged black holes in five dimensions, as solutions of a gauged supergravity theory, were obtained in [6]. These, and some higher-dimensional generalisations obtained in [7], were all non-rotating.

Charged rotating black holes in four dimensional theories with a cosmological constant were obtained long ago [8], but until recently no analogous five-dimensional charged rotating solutions were known, except in certain extremal BPS limiting cases [9, 10]. In a recent paper [11], we constructed general solutions for charged rotating black holes in five-dimensional gauged  $\mathcal{N} = 2$  pure supergravity, in the case where the two angular momenta are taken to be equal. These non-extremal solutions encompass the extremal solutions of [9, 10] as special cases. By instead setting the charge to zero, the solutions in [11] reduce to the rotating five-dimensional Kerr-de Sitter black holes of [12], in the special case where the two rotation parameters are set equal.

In this letter we extend our previous results, by constructing a general class of non-extremal charged rotating black hole solutions in the five-dimensional  $U(1)^3$  gauged theory of  $\mathcal{N} = 2$  supergravity coupled to two vector multiplets. We obtain the general non-extremal solutions of this dilatonic theory, with three independent electric charges, subject to the specialisation that the two angular momenta in the orthogonal 4-space are set equal. These 3-charge solutions are important for probing fully the microscopic degrees of freedom associated with the three R-charges in the dual  $\mathcal{N} = 4$  CFT on the boundary, without the loss of information that would be inherent if the three charges were set equal.

Our new 3-charge solutions are generalisations to the gauged theory of the 3-charge spinning black hole solutions (with two rotation parameters set equal) of the corresponding five dimensional *ungauged* supergravity, obtained in [13]. They also, of course, specialise to our previous results in [11] if one sets the three electric charges equal, under which circumstance the two dilatonic scalars decouple and become constant.

The bosonic sector of the five-dimensional  $\mathcal{N} = 2$  gauged supergravity coupled to two vector multiplets is described by the Lagrangian

$$e^{-1} \mathcal{L} = R - \frac{1}{2} \partial \vec{\varphi}^2 - \frac{1}{4} \sum_{i=1}^3 X_i^{-2} (F^i)^2 - \lambda \sum_{i=1}^3 X_i^{-1} + \frac{1}{24} \epsilon_{ijk} \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu}^i F_{\rho\sigma}^j A_{\lambda}^k, \quad (1)$$

where  $\vec{\varphi} = (\varphi_1, \varphi_2)$ , and

$$X_1 = e^{-\frac{1}{\sqrt{6}}\varphi_1 - \frac{1}{\sqrt{2}}\varphi_2}, \quad X_2 = e^{-\frac{1}{\sqrt{6}}\varphi_1 + \frac{1}{\sqrt{2}}\varphi_2}, \quad X_3 = e^{\frac{2}{\sqrt{6}}\varphi_1}. \quad (2)$$

The gauge-coupling constant  $g$  is related to  $\lambda$  by  $\lambda = -g^2$ .

The solutions that we have obtained are as follows:

$$ds_5^2 = -\frac{Y-f_3}{R^2}dt^2 + \frac{r^2 R}{Y}dr^2 + R d\Omega_3^2 + \frac{f_1-R^3}{R^2}(\sin^2 \theta d\phi + \cos^2 \theta d\psi)^2 - \frac{2f_2}{R^2}dt(\sin^2 \theta d\phi + \cos^2 \theta d\psi), \quad (3)$$

$$A^i = \frac{\mu}{r^2 H_i} \left( s_i c_i dt + \ell (c_i s_j s_k - s_i c_j c_k) (\sin^2 \theta d\phi + \cos^2 \theta d\psi) \right), \quad (4)$$

$$X_i = \frac{R}{r^2 H_i}, \quad i = 1, 2, 3 \quad (5)$$

where

$$R \equiv r^2 \left( \prod_{i=1}^3 H_i \right)^{\frac{1}{3}}, \quad H_i \equiv 1 + \frac{\mu s_i^2}{r^2}, \quad d\Omega_3^2 = d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\psi^2, \quad (6)$$

and  $s_i$  and  $c_i$  are shorthand notations for

$$s_i \equiv \sinh \delta_i, \quad c_i \equiv \cosh \delta_i, \quad i = 1, 2, 3. \quad (7)$$

Note that in the expressions (4) for the vector potentials  $A^i$ , the triplet indices  $(i, j, k)$  are all unequal:  $(i \neq j \neq k \neq i)$ . The functions  $(f_1, f_2, f_3, Y)$  are given by

$$\begin{aligned} f_1 &= R^3 + \mu \ell^2 r^2 + \mu^2 \ell^2 \left[ 2 \left( \prod_i c_i - \prod_i s_i \right) \prod_j s_j - \sum_{i < j} s_i^2 s_j^2 \right], \\ f_2 &= \gamma \ell \lambda R^3 + \mu \ell \left( \prod_i c_i - \prod_i s_i \right) r^2 + \mu^2 \ell \prod_i s_i, \\ f_3 &= \gamma^2 \ell^2 \lambda^2 R^3 + \mu \ell^2 \lambda \left[ 2 \gamma \left( \prod_i c_i - \prod_i s_i \right) - \Sigma \right] r^2 \\ &\quad + \mu \ell^2 - \lambda \Sigma \mu^2 \ell^2 \left[ 2 \left( \prod_i c_i - \prod_i s_i \right) \prod_j s_j - \sum_{i < j} s_i^2 s_j^2 \right] + 2 \lambda \gamma \mu^2 \ell^2 \prod_i s_i, \\ Y &= f_3 - \lambda \Sigma R^3 + r^4 - \mu r^2, \end{aligned} \quad (8)$$

where

$$\Sigma \equiv 1 + \gamma^2 \ell^2 \lambda. \quad (9)$$

It is helpful to note that  $\sqrt{-g}$  takes a simple form, namely  $\sqrt{-g} = r R \sin \theta \cos \theta$ .

We arrived at the above solution by making conjectures for the expressions for the metric, vector potentials and dilatonic scalars that reduced to previously-known cases under

appropriate limits. In particular, we were guided by the results for the ungauged case in [13], and the results for the non-dilatonic gauged case (i.e. with three equal charges) in [11]. Verifying that the conjectured configuration solves the equations of motion following from (1) is then a straightforward mechanical exercise, which is most easily accomplished with the aid of a computer. (We used Mathematica for this purpose.) There are six free parameters in the solution, namely  $(\mu, \delta_1, \delta_2, \delta_3, \ell, \gamma)$ . The constant  $\mu$ , together with the three “non-extremality parameters”  $\delta_i$ , characterise the mass and the three electric charges associated with the three vector potentials  $A^i$ . The parameter  $\ell$  characterises the rotation of the black hole. One can define “physical” mass, charge and angular momentum parameters  $M$ ,  $Q_i$  and  $J$ , according to

$$\begin{aligned} M &= \frac{1}{2}\mu \sum_i (s_i^2 + c_i^2), \\ Q_i &= \mu s_i c_i, \\ J &= \mu \ell \left( \prod_i c_i - \prod_i s_i \right). \end{aligned} \tag{10}$$

The sixth constant,  $\gamma$ , is the analogue of the extra parameter  $\beta$  found in the charged rotating black holes of the pure  $\mathcal{N} = 2$  gauged supergravity theory in [11]. It is in fact a trivial parameter (see [11], and, for a complete proof of its triviality, [14]). It is nonetheless useful to retain the redundant parameter  $\gamma$ , since it provides a convenient way to consider various limits.

In order to make the global structure of the metrics more apparent, it is convenient to rewrite the metric (3) in terms of left-invariant 1-forms  $\sigma_i$  on  $S^3$ . Defining

$$\begin{aligned} \sigma_1 &= \cos \tilde{\psi} d\tilde{\theta} + \sin \tilde{\psi} \sin \tilde{\theta} d\tilde{\phi}, \\ \sigma_2 &= -\sin \tilde{\psi} d\tilde{\theta} + \cos \tilde{\psi} \sin \tilde{\theta} d\tilde{\phi}, \\ \sigma_3 &= d\tilde{\psi} + \cos \tilde{\theta} d\tilde{\phi}, \end{aligned} \tag{11}$$

where

$$\psi - \phi = \tilde{\phi}, \quad \psi + \phi = \tilde{\psi}, \quad \theta = \frac{1}{2}\tilde{\theta}, \tag{12}$$

we find that (3) can be rewritten as

$$ds_5^2 = -\frac{RY}{f_1} dt^2 + \frac{r^2 R}{Y} dr^2 + \frac{1}{4}R(\sigma_1^2 + \sigma_2^2) + \frac{f_1}{4R^2}(\sigma_3 - \frac{2f_2}{f_1} dt)^2, \tag{13}$$

whilst the vector potentials in (4) become

$$A^i = \frac{\mu}{r^2 H_i} \left( s_i c_i dt + \frac{1}{2}\ell (c_i s_j s_k - s_i c_j c_k) \sigma_3 \right). \tag{14}$$

## Reductions to Previously-known Solutions

Various limits of our new solutions reduce to previously-known cases. These include the non-extremal 3-charge spinning black hole solutions of the ungauged theory in [13] (specialised to the case of equal angular momenta); the non-extremal charged rotating solutions of the pure  $\mathcal{N} = 2$  gauged theory found recently in [11]; the BPS 3-charge rotating solutions of Klemm and Sabra [15]; and the BPS 3-charge rotating solutions of Gutowski and Reall [16]. In detail, these various cases arise as follows:

- The ungauged limit (i.e.  $\lambda = 0$ ) leads to the special case of the solutions of [13] where one sets the two angular momenta parameters equal, i.e.  $\ell_1 = \ell_2 = \ell$ . This limit is obtained from our solution by setting  $\lambda = 0$ , replacing  $\mu$  by  $2m$ , and by redefining  $r^2 \rightarrow r^2 + \ell^2$ .

- The equal-charge limit, i.e. setting  $\delta_1 = \delta_2 = \delta_3 \equiv \delta$ , reduces to the solution found in [11]. Note that the parameters  $\beta$  and  $J$  in [11] are related to  $\gamma$  and  $\ell$  of the present paper by

$$\beta = \gamma e^\delta, \quad J = \ell e^{-\delta}, \quad (15)$$

while the parameters  $M$  and  $Q$  in [11] can be read off from (10). The radial variable in [11] is given by sending  $(r^2 + \mu \sinh^2 \delta) \rightarrow r^2$ .

- The Klemm-Sabra solution [15], which has closed timelike curves, is a BPS limit of our solution, obtained by taking

$$\begin{aligned} \mu &\rightarrow 0, & \delta_i &\rightarrow -\infty, \\ Q_i &= \frac{1}{4}\mu e^{-2\delta_i}, & \ell &= \alpha \sqrt{\mu}, \end{aligned} \quad (16)$$

where the three charges  $Q_i$  and the constant  $\alpha$  are kept finite and non-zero. The black hole mass and angular momentum, defined in (10), are then given by

$$M = -(Q_1 + Q_2 + Q_3), \quad J = 2\alpha \sqrt{Q_1 Q_2 Q_3}. \quad (17)$$

Note that since  $\ell \rightarrow 0$  and the  $\gamma$  parameter appears only in a product with  $\ell$ , the solution does not depend on  $\gamma$ .

- The Gutowski-Reall solution [16] is a regular BPS limit of our solution, obtained by taking

$$\begin{aligned} \mu &\rightarrow 0, & \delta_i &\rightarrow +\infty, \\ Q_i &= \frac{1}{4}\mu e^{2\delta_i}, & \gamma &= \sqrt{\mu}, & \ell &= \frac{1}{\sqrt{-\lambda \mu}}, \end{aligned} \quad (18)$$

where the charges  $Q_i$  are kept finite and non-zero. Note that  $\gamma$  goes to zero, while  $\ell$  goes to infinity, when the limit is taken. (The solution remains finite, however.) The black hole mass and the angular momentum, following from (10), are:

$$M = +(Q_1 + Q_2 + Q_3), \quad J = \frac{Q_1 Q_2 + Q_1 Q_3 + Q_2 Q_3}{2\sqrt{-\lambda} \sqrt{Q_1 Q_2 Q_3}}. \quad (19)$$

### Further Remarks

The metric (13) has horizons at values of the radial coordinate where  $RY f_1^{-1}$  vanishes. In order to avoid naked singularities, the outer horizon at  $r = r_H$  should lie outside the curvature singularity at  $R = 0$ , and thus we require that it occur at the largest positive root of  $Y(r_H) = 0$ . In order to avoid having closed timelike curves (CTCs),  $f_1$  should be positive for all  $r > r_H$ . A detailed analysis of the restrictions on the parameters in order to obtain solutions free of naked singularities or closed CTCs is quite involved, and we shall not present it here. It is analogous to the one given in [11] for the case where all three charges are equal. Clearly, there exist appropriate ranges of the parameters for which such “regular” black holes arise.

On the horizon, the Killing vector

$$l = \frac{\partial}{\partial t} - \frac{2f_2(r_H)}{f_1(r_H)} \frac{\partial}{\partial \psi} \quad (20)$$

becomes null, and thus  $r = r_H$  corresponds to a Killing horizon. The surface gravity  $\kappa$ , which is constant over the horizon, can be calculated from

$$\kappa^2 = (\partial_\mu K) (\partial^\mu K) \Big|_{r=r_H}, \quad (21)$$

where  $K = \sqrt{-l^\mu l_\mu}$ , implying that

$$\kappa = \left| \frac{Y'(r_H)}{4r_H^2 f_1(r_H)} \right|. \quad (22)$$

The area of the Killing horizon is given by

$$A = 2\pi^2 \sqrt{f_1(r_H)}. \quad (23)$$

The Hawking temperature and entropy are therefore given by

$$T = \left| \frac{Y'(r_H)}{8\pi r_H^2 f_1(r_H)} \right|, \quad S = \frac{1}{2} \pi^2 \sqrt{f_1(r_H)}. \quad (24)$$

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